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Ch-5

Arithmetic Progression

★ Important Points :-

1. Sequence: A set of numbers arranged in some definite order and formed according to some rules is called sequence

2. Arithmetic Progression: A sequence in which the difference of each term from its succeding term is constant throughout, is called an arithmetic sequence or arithmetic progression (A.P.)
In other words A.P. is sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ such that $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots a_n - a_{n-1} = d$ and so on.

3. General Term: If 'a' is the first tem and 'd' is common difference in an A.P., then n^{th} term (general tem) is given by $a_n = a+(n-1)d$.

4. Sum of n terms of an A.P.: If 'a' is the first term and 'd' is the common difference of an A.P., then sum of first n terms is given by

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

If 'a' is the first term & 'l' is the last / n^{th} term of finite A.P., then the sum is given by $S_n = \frac{n}{2}\{a+l\}$

5.(i) If a_n is given, then common difference $d = a_n - a_{n-1}$
(ii) If S_n is given, then n^{th} term is given by $a_n = S_n - S_n - 1$
(iii) If a, b, c are in A.P., then $2b = a+c$
(v) Difference of m^{th} and n^{th} term of an A.P. $= (m-n)d$.
(iv) If a sequence has n terms, its r^{th} term from the end $= (n-r+1)^{th}$ term from the beginning.

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* Exercise - 5.1

Q:1) Find the 1^{st} term a and common difference d for following AP.

(i) 6, 9, 12, 15...

Ans $a_1 = 6$; common difference $(d) = a_2 - a_1 = 9-6$
$= 3$

$a = 6$, $d = 3$ Ans

(ii) −7, −9, −11, −13...

Ans $a_1 = -7$; $d = a_2 - a_1 = -9-(-7)$
$= -9+7 = -2$

$a = -7$, $d = -2$ Ans

(iii) $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2} \ldots \Rightarrow a_1 = \frac{3}{2}$; $d = \overset{a_2}{\frac{1}{2}} - \overset{a_1}{\frac{3}{2}} = \frac{-\cancel{2}}{\cancel{2}} = -1$

$a = 3/2$, $d = -1$ Ans

(iv) $1, -2, -5, -8 \ldots \Rightarrow a_1 = 1$, $d = a_2 - a_1$
$= -2-1 = -3$

$a = 1$, $d = -3$ Ans

(v) $-1, \frac{1}{4}, \frac{3}{2} \ldots \Rightarrow a_1 = -1$, $d = a_2 - a_1$
$= \frac{1}{4} - (-1)$
$= \frac{1}{4} + \frac{1}{1} = \frac{1+4}{4} = \frac{5}{4}$

$a = -1$, $d = 5/4$ Ans

(vi) $3, 1, -1, -3 \ldots \Rightarrow a_1 = 3$, $d = 1-3$
$= -2$

$a = 3$, $d = -2$ Ans

(vii) $3, -2, -7, -2 \ldots \Rightarrow a_1 = 3, \quad d = -2 - 3$
$= -5$

$a = 3, \; d = -5$

Q:2) If first term (a) & common difference (d) of A.P. is given the find the first four term of that progression.

(i) $a = -1, \; d = 1/2$

Ans $a_1 = -1$

$a_2 = a_1 + d = -1 + 1/2$
$= \frac{-2+1}{2} = -1/2$

$a_3 = a + 2d = \cancel{-1 + 2/2} = -1 + \cancel{2} \times \frac{1}{\cancel{2}} = -1 + 1 = 0$
$= \cancel{\frac{-2+2}{2}}$

$a_4 = a + 3d = -1 + 3 \times 1/2 = -1 + 3/2$
$= \frac{-2+3}{2} = \frac{1}{2}$

$-1, -1/2, 0, 1/2$. Ans. ← first four terms of A.P.

(ii) $a = 1/3, \; d = 4/3$

Ans $a_1 = 1/3$

$a_2 = a_1 + d = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$

$a_3 = a + 2d = \frac{1}{3} + 2 \times \frac{4}{3} = \frac{1}{3} + \frac{8}{3} = \frac{9}{3}$

$a_4 = a + 3d = \frac{1}{3} + \frac{3 \times 4}{3} = \frac{1}{3} + \frac{12}{3} = \frac{13}{3}$

$\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{12}{3}$ Ans

(iii) $a = 0.6$, $d = 1.1$

Ans $a_1 = 0.6$

$a_2 = a + d = 0.6 + 1.1 = 1.7$

$a_3 = a + 2d = 0.6 + 2 \times 1.1 = 2.8$

$a_4 = a + 3d = 0.6 + 3 \times 1.1 = 3.9$

0.6, 1.7, 2.8, 3.9. Ans.

(iv) $a = 4$, $d = -3$

Ans. $a_1 = 4$

$a_2 = a + d = 4 + (-3) = 1$

$a_3 = a + 2d = 4 + 2(-3) = 4 - 6 = -2$

$a_4 = a + 3d = 4 + 3(-3) = 4 - 9 = -5$

4, 1, −2, −5. Ans.

(v) $a = 11$, $d = -4$

Ans $a_1 = 11$

$a_2 = a + d = 11 + (-4) = 7$

$a_3 = a + 2d = 11 + 2(-4) = 11 - 8 = 3$

$a_4 = a + 3d = 11 + 3(-4) = 11 - 12 = -1$

11, 7, 3, −1. Ans.

(vi) $a = -1.25$, $d = -0.25$

Ans. $a_1 = -1.25$

$a_2 = a + d = -1.25 + (-0.25) = -1.50$

$a_3 = a + 2d = -1.25 + 2(-0.25) = -1.75$

$a_4 = a + 3d = -1.25 + 3(-0.25) = -2.00$

−1.25, −1.50, −1.75, −2.00 Ans.

(vii) $a = 20$, $d = -3/4$

Ans $a_1 = 20$

$a_2 = a+d = 20+(-3/4) =$

$= \frac{80-3}{4} = \frac{77}{4}$

$a_3 = a+2d = \cancel{20+2(-3/4)} = \cancel{20 - 3} = 20+2(-3/4)$

$= 20 - 6/4 = \frac{80-6}{4} = \frac{74}{4}$

$a_4 = a+3d = 20+3(-3/4) = 20 - 9/4$

$= \frac{80-9}{4} = \frac{71}{4}$

$20, 77/4, 74/4, 71/4$. Ans.

Q.3. Test A.P. for given series of numbers. For an A.P. find its common difference and next 4 terms.

(i) $2, 5/2, 3, 7/2, \ldots$

Ans.) ~~common~~ difference between 2 consecutive terms (d)

$d = a_2 - a_1 \Rightarrow \frac{5}{2} - \frac{2}{1} \Rightarrow \frac{5-4}{2} = \frac{1}{2}$

$d = a_3 - a_2 \Rightarrow \frac{3}{1} - \frac{5}{2} \Rightarrow \frac{6-5}{2} = \frac{1}{2}$

$d = a_4 - a_3 \Rightarrow \frac{7}{2} - \frac{3}{1} \Rightarrow \frac{7-6}{2} = \frac{1}{2}$

common difference (d) $= \frac{1}{2}$

Next four terms $\Rightarrow a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$

$a_6 = \frac{8}{2} + \frac{1}{2} = \frac{9}{2}$; $a_7 = \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$; $a_8 = \frac{10}{2} + \frac{1}{2} = \frac{11}{2}$

$\therefore 4, \frac{9}{2}, 5, \frac{11}{2}$ are next four terms.

(ii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

Ans. $d \Rightarrow a_2 - a_1 = -\frac{1}{2} - (-\frac{1}{2}) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - (-\frac{1}{2}) = -\frac{1}{2} + \frac{1}{2} = 0$

Here, $d = 0$

$a_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_6 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_8 = -\frac{1}{2} + 0 = -\frac{1}{2}$.

These are next 4 terms.

(iii) $a, a^2, a^3, a^4 \ldots$

Ans.) $d \Rightarrow a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a-1)$

Here common difference is not similar

So, given series is not an A.P.

(iv) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

Ans.) $d \Rightarrow a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = \sqrt{3}(\sqrt{4} - \sqrt{3})$

Here, common difference is not same

So, given series is not an A.P.

(v) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

Ans.) $d \Rightarrow a_2 - a_1 = \sqrt{8} - \sqrt{2} = \sqrt{2}(\sqrt{4} - 1) = \sqrt{2}(2-1) = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = \sqrt{2}(\sqrt{9} - \sqrt{4}) = \sqrt{2}(3-2) = \sqrt{2}$

$a_4 - a_2 = \sqrt{32} - \sqrt{18} = \sqrt{2}(\sqrt{16} - \sqrt{9}) = \sqrt{2}(4-3) = \sqrt{2}$

Here, $d = \sqrt{2}$ & it is same

So, given series is an A.P.

Then next 4 terms are

$a_5 = a_4 + d = \sqrt{32} + \sqrt{2} = \sqrt{2}(\sqrt{16}+1) = \sqrt{2}(4+1) = \sqrt{2}(5)$
$= \sqrt{2} \times \sqrt{25}$
$= \sqrt{50}$

$a_6 = a_5 + d = \sqrt{50} + \sqrt{2} = \sqrt{2}(\sqrt{25}+1) = \sqrt{2}(5+1) = \sqrt{2}(6)$
$= \sqrt{2} \times (\sqrt{36})$
$= \sqrt{72}$

$a_7 = a_6 + d = \sqrt{72} + \sqrt{2} = \sqrt{2}(\sqrt{36}+1) = \sqrt{2}(6+1) = \sqrt{2}(7)$
$= \sqrt{2} \times \sqrt{49}$
$= \sqrt{98}$

$a_8 = a_7 + d = \sqrt{98} + \sqrt{2} = \sqrt{2}(\sqrt{49}+1) = \sqrt{2}(7+1) = \sqrt{2}(8)$
$= \sqrt{2} \times \sqrt{64}$ 04
$= \sqrt{128}$

$\sqrt{50}, \sqrt{72}, \sqrt{98}, \sqrt{128}$ are next 4 terms & common difference (d) is $\sqrt{2}$.

(vi) $a, 2a, 3a, 4a \ldots$

Ans $d \Rightarrow a_2 - a_1 = 2a - a = a$
$a_3 - a_2 = 3a - 2a = a$
$a_4 - a_3 = 4a - 3a = a$
$d = a$ i.e. same, So it is an A.P.
Then next four terms are
$a_5 = a_4 + d = 4a + a = 5a$
$a_6 = a_5 + d = 5a + a = 6a$
$a_7 = a_6 + d = 6a + a = 7a$
$a_8 = a_7 + d = 7a + a = 8a$

(vii) $0.2, 0.22, 0.222, 0.2222, \ldots$

Ans) $d \Rightarrow a_2 - a_1 = 0.22 - 0.2 = 0.02$
$a_3 - a_2 = 0.222 - 0.22 = 0.002$
$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$
Here, (d) is not similar, So given series is not an A.P.

(viii) $3,\ 3+\sqrt{2},\ 3+2\sqrt{2},\ 3+3\sqrt{2}\ \ldots\ldots$

Ans) $a_2 - a_1 = \cancel{3}+\sqrt{2} - \cancel{3} = \sqrt{2}$

$a_3 - a_2 = \cancel{3}+2\sqrt{2} - (\cancel{3}+\sqrt{2}) = \sqrt{2}$

$a_4 - a_3 = \cancel{3}+3\sqrt{2} - (\cancel{3}+2\sqrt{2}) = \sqrt{2}$

Here, $d = \sqrt{2}$ i.e. same, so its next 4 terms are,

$a_5 = a_4 + d = 3+3\sqrt{2} + \sqrt{2} = 3+4\sqrt{2}$

$a_6 = a_5 + d = 3+4\sqrt{2} + \sqrt{2} = 3+5\sqrt{2}$

$a_7 = a_6 + d = 3+5\sqrt{2} + \sqrt{2} = 3+6\sqrt{2}$ — Ans.

$a_8 = a_7 + d = 3+6\sqrt{2} + \sqrt{2} = 3+7\sqrt{2}$

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Ex :- 5.2

Q:1.) Find :

(i) 10^{th} term of A.P. $2, 7, 12 \ldots$

Ans) $a = 7$

$d \Rightarrow a_2 - a_1 = 7-2 = 5.$

$n = 10$

$a_n = a + (n-1) \times d$

$a_{10} = 2 + (10-1) \times 5$

$a_{10} = 2 + 9 \times 5$

$a_{10} = 2 + 45$

$a_{10} = 47$ Ans.

(ii) 18^{th} term of A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2} \ldots$

Ans) $a = \sqrt{2}$

$d \Rightarrow 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$n = 18$

$a_n = a + (n-1) \times d$

$a_{18} = \sqrt{2} + (18-1) \times 2\sqrt{2}$

$a_{18} = \sqrt{2} + 17 \times 2\sqrt{2}$

$a_{18} = \sqrt{2} + 34\sqrt{2}$

$a_{18} = 35\sqrt{2}$ Ans.

(iii) 24^{th} term of A.P. 9, 13, 17, 21 ....

Ans) $a = 9$

$d = 13 - 9 \; (a_2 - a_1) = 4$

$n = 24$

$a_n = a + (n-1) \times d$

$a_{24} = 9 + (24-1) \times 4$

$a_{24} = 9 + 23 \times 4$

$a_{24} = 9 + 92$

$a_{24} = 101$ Ans.

Q. 2. Solve:

(i) Which term of A.P. 21, 18, 15 .... is -81?

Ans) $a = 21$

$d = a_2 - a_1 = 18 - 21 = -3$

$a_n = 81 \quad (n = ?)$

$a_n = a + (n-1) \times d$

$-81 = 21 + (n-1) \times -3$

$-81 = 21 - 3n + 3$

$-81 - 24 = -3n$

$-105 = -3n$

$n = \frac{\cancel{+105}^{35}}{\cancel{+3}_{1}}$

$n = 35.$

Hence, -81 is 35^{th} term of given A.P.

(ii) Which term of A.P 84, 80, 76 ..... is 0?

Ans. $a = 84$

$d = a_2 - a_1 = 80 - 84 = -4$

$a_n = 0 \quad (n = ?)$

$a_n = a + (n-1) \times d$

$0 = 84 + (n-1) \times -4$

$0 - 84 = -4n + 4$

$-84 - 4 = -4n$

$-88 = -4n$

$n = \frac{\cancel{+88}^{22}}{\cancel{+4}_1}$

$n = 22$

Hence, 0 is 22^{th} term of given A.P.

(iii) Is 301 any term of series 5, 11, 17, 23 ....?

Ans) $a = 5$

$d = 11 - 5 \; (a_2 - a_1) = 6$

$a_n = a + (n-1) \times d$

$301 = 5 + (n-1) \times 6$

$\frac{301 - 5}{6} = n - 1$

$n - 1 = \frac{296}{6}$

$n - 1 = 49.33$

$n = 49.33 + 1$

$n = 50.33$

Here, n is not a natural number. So no term can be 301 in given AP.

(iv) Is -150 is any term of A.P. $11, 8, 5, 2 \ldots$?

Ans.) $a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

$a_n = a + (n-1) \times d$

$-150 = 11 + (n-1) \times -3$

$-150 - 11 = (n-1) \times -3$

$-161 = (n-1) \times -3$

$n - 1 = \frac{\cancel{-}161}{\cancel{-}3}$

$n - 1 = 53.6$ (Approx)

$n = 53.6 + 1$

$n = 54.6$

Here n is not a natural no. So -150 is not ~~a~~ term of given A.P.

Q.3) If 6^{th} term and 17^{th} term of A.P. are 19 and 41 respectively, then find 40^{th} term.

Ans.) $a_6 = 19$; $a_{17} = 41$ $a_{40} = ?$

$a_6 = a + 5d = 19$ — ①

$a_{17} = a + 16d = 41$ — ②

Subtract:- $-11d = -22$

$d = \frac{22}{11} = 2.$

Put value of $d = 2$ in eq. 1

$a + 5 \times 2 = 19$

$a + 10 = 19$

$a = 19 - 10$

$a = 9.$

$a_{40} = a + (n-1) \times d$
$a_{40} = 9 + 39 \times 2$
$a_{40} = 9 + 78$
$a_{40} = 87$ Ans.

Q:4. Third and ninth term of an A.P. are 4 and -8 respectively then its which term will be 0 ?

Ans) $a_3 = 4$; $a_9 = -8$

$a_3 = \cancel{a} + 2d = 4$ —— ①
$a_9 = \cancel{a} + 8d = -8$ —— ②

Substract:- $-6d = 12$

$d = \frac{-12}{6}$

$d = -2$

Put value of $d = -2$ in eq 1

$a + 2 \times (-2) = 4$
$a - 4 = 4$
$a = 4 + 4$
$a = 8$

Let n^{th} term of series will be 0, then $a_n = 0$

$a_n = a + (n-1) \times d$
$0 = 8 + (n-1) \times -2$
$\frac{+\cancel{8}^{4}}{+\cancel{2}_{1}} = n - 1$
$n - 1 = 4$
$n = 4 + 1$
$n = 5$, Hence 5^{th} term term is 0.

Q:5. 3rd term of an A.P. is 16 and 7^{th} term is 12 more than 5^{th} term then find A.P.

Ans) Given :- $a_3 = 16$

$a_3 = a + 2d = 16$ — ①

A.T.Q.

$a_7 - a_5 = 12$

$\cancel{a} + 6d - \cancel{a} - 4d = 12$

$2d = 12$

$d = \frac{12}{2} = 6$

Put value of $d = 6$ in eq 1

$a + 2 \times 6 = 16$

$a + 12 = 16$

$a = 16 - 12$

$a = 4$

$a = 4$; $d = 6$

A.P. = $a, a+d, a+2d$

$4, 4+6, 4+2\times6$

$4, 10, 16$

So required A.P. is 4, 10, 16 .... Ans.

Q: 6. How many three digit no. are divisible by 7?

Ans :- Series of 3 digit no. 100, 101 999

First three digit divisible by 7 = 105, 112, 119 994

Let total no. of terms is n.

$a_1 = 105$; $d = 7$; $a_n = 994$

$$a_n = a + (n-1) \times d$$

$$994 = 105 + (n-1) \times 7$$

$$\frac{994 - 105}{7} = n - 1$$

$$\frac{889}{7} = n - 1$$

$$n = 127 + 1$$

$$n = 128 \text{ Ans.}$$

So, there are 3 digit no. divisible by 7 is 128.

Q:7) Find the 11^{th} term from last of A.P 10, 7, 4 ..... −62 ?

Ans. $a_1 = 10$; $d = -3$; $l = -62$

$$a_n = l - (n-1) \times d$$

$$a_{11} = l - 10d$$

$$a_{11} = -62 (-10 \times -3)$$

$$a_{11} = -62 + 30$$

$$a_{11} = -32 \text{ Ans.}$$

Q:8) Find the 12^{th} term from last of A.P. 1, 4, 7, 10 .... 88 ?

Ans. $a_1 = 1$; $d = 3$; $l = 88$

$$a_{12} = 88 - 11 \times 33$$

$$a_{12} = 88 - 33$$

$$a_{12} = 55 \text{ Ans}$$

Q:9) There are 60 terms in an A.P. If its first and last term are 7 and 125 respectively, then find its 32nd term.

Ans) $n = 60$; $a_1 = 7$, $a_n = 125$

$a_{32} = ?$

$a_n = a + (n-1) \times d$

$125 = 7 + (60-1) \times d$

$125 - 7 = 59 \times d$

$\frac{118}{59} = d$

$d = 2$ Ans.

$a_{32} = a + 31d$

$a_{32} = 7 + 31 \times 2$

$a_{32} = 7 + 62$

$a_{32} = 69$ Ans.

Q:10 Four numbers are in A.P. If sum of no.s is 50 & larger no. is 4 times the smaller no. then find the no.s.

Ans.) Let four no. of A.P. are –

$a,\ a+d,\ a+2d,\ a+3d$

A.T.Q.

$a + a + d + a + 2d + a + 3d = 50$

$4a + 6d = 50$

$2(2a + 3d) = 50$

$2a + 3d = 50/2$

$2a + 3d = 25$ — ①

If larger no. is 4 times the smaller than

$a + 3d = 4a$

$3d = 4a - a$

$\cancel{3}d = \cancel{3}a$

$d = a$ — ②

From eq ① & ②

$2a + 3d = 25$ $(d = a)$

$2a + 3a = 25$

$5a = 25$

$a = 25/5$

$a = 5$

$(a = d)$ $\therefore$ $d = 5$

$a_1 = 5$

$a_2 = 10$

$a_3 = 15$

$a_4 = 20$

Ans. So, 4 no.s. are 5, 10, 15, 20

* Exercise 5.3

q:1) Find the sum of following A.P.

(i) 1, 3, 5, 7 upto 12 terms.

Ans. $a_1 = 1$; $d = 2$

Sum of n term $(S_n) = n/2\,[2a + (n-1) \times d]$

$$S_{12} = \frac{12}{2}[2 \times 1 + (12-1) \times 2]$$

$$S_{12} = 6\,[2 + (11 \times 2)]$$

$$S_{12} = 6\,[2 + 22]$$

$$S_{12} = 6 \times 24$$

$$S_{12} = 144 \quad \text{Ans.}$$

(ii) 8, 3, −2 .... upto 22 terms

Ans. $a_1 = 8$; $d = -5$

$$S_{22} = \frac{n}{2}[2 \times 1 + (n-1) \times d$$

$$S_{22} = \frac{22}{2}\left[2\times 8 + (22-1)\times(-5)\right]$$

$$S_{22} = 11\left[16 + 21\times(-5)\right]$$

$$S_{22} = 11\left[16 - 105\right]$$

$$S_{22} = 11 \times -89$$

$$S_{22} = -979$$

(iii) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ ..... upto 11 terms.

Ans.) $a_1 = \frac{1}{15}$; $d = \frac{1}{60}$

$$S_{11} = \frac{n}{2}\left[2a + (n-1)\times d\right]$$

$$S_{11} = \frac{11}{2}\left[2\times\frac{1}{15} + (11-1)\times\frac{1}{60}\right]$$

$$S_{11} = \frac{11}{2}\left[\frac{2}{15} + 10\times\frac{1}{60}\right]$$

$$S_{11} = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$S_{11} = \frac{11}{2}\times\frac{\cancel{9}\,3}{\cancel{30}\,10}$$

$$S_{11} = \frac{33}{20} \text{ Ans.}$$

Q:2) Find the sum of the following:

(i) $3 + 11 + 19 + \ldots + 803$

Ans.) $a_1 = 3$; $d = 11 - 3 = 8$; $a_n = 803$

$n \Rightarrow \quad a_n = a + (n-1)\times d$

$$803 = 3 + (n-1)\times 8$$

$\frac{803-3}{8} = n-1$

$\frac{800}{8} = n-1$

$100+1 = n$

$\underline{n = 101}$

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{101}{2}(3+803)$

$S_n = \frac{101}{\cancel{2}_1} \times \cancel{806}^{403}$

$S_n = 40703$ Ans.

(ii) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

Ans.) $a_1 = 7$; $d = 21/2 - 7 = 7/2$; $a_n = 84$

$n \Rightarrow$ $a_n = a + (n-1) \times d$

$84 = 7 + (n-1) \times 7/2$

$84 - 7 = (n-1) \times 7/2$

$\cancel{77}^{11} \times \frac{2}{\cancel{7}_1} = n-1$

$n = 22+1$

$n = 23$

$S_n = n/2 (a+l) \Rightarrow S_n = 23/2 (7+84)$

$S_n = 23/2 (91)$

$S_n = \frac{2093}{2}$

$S_n = 1046\frac{1}{2}$ Ans

Q:3 Find the no. of terms.

(i) How many terms of A.P. 9, 17, 25.... taken so that their sum is 636 ?

Ans. $a_1 = 9$; $d = 17 - 9 = 8$; $S_n = 636$ [$n = ?$]

$$S_n = \frac{n}{2}[2a + (n-1) \times d]$$

$$636 = n/_2 [2 \times 9 + (n-1) \times 8]$$

$$636 \times 2 = n[18 + 8n - 8]$$

$$1272 = n[10 + 8n]$$

$$1272 = 10n + 8n^2$$

$$8n^2 + 10n = 1272$$

$$2(4n^2 + 5n) = 1272$$

$$4n^2 + 5n = 1272/2$$

$$4n^2 + 5n = 636$$

$$4n^2 + 5n - 636 = 0$$

$$4n^2 + 53n - 48n - 636 = 0$$

$$n(4n + 53) - 12(4n + 53) = 0$$

$$(4n + 53)(n - 12) = 0$$

$$4n + 53 = 0 \quad ; \quad n - 12 = 0$$

$$n = \frac{-53}{4} \ (\times) \quad ; \quad n = 12 \ (\checkmark)$$

Hence, $n = 12$ Ans

(ii) How many terms of A.P. 63, 60, 57 .... taken to give a sum of 693 ?

Ans. $a_1 = 63$; $d = -3$; $S_n = 693$ [$n = ?$]

$$S_n = \frac{n}{2}[2a + (n-1) \times d]$$

$693 = n/2\,[2 \times 63 + (n-1) \times (-3)]$
$693 \times 2 = n\,[126 - 3n + 3]$
$1386 = n\,[129 - 3n]$
$1386 = 129n - 3n^2$
$-3n^2 + 129n = 1386$
$-3(n^2 - 43n) = 1386$
$n^2 - 43n = 1386 / -3$
$n^2 - 43n = -462$
$n^2 - 43n + 462 = 0$
$n^2 - 21n - 22n + 462 = 0$
$n(n-21) - 22(n-21) = 0$
$(n-21)(n-22) = 0$
$n - 21 = 0 \;;\; n - 22 = 0$
$n = 21$ (✓) ; $n = 22$ (✓)

Hence, n can be 21 or 22 Ans.

Q:4) Find the sum of first 25 terms of following series whose n^{th} term is given :

(i) $a_n = 3 + 4n$ — ①

Ans) Substituting various values of n in eq ①

$a_1 = 3 + 4(1) = 7$
$a_2 = 3 + 4(2) = 11$
$a_3 = 3 + 4(3) = 15$
$d = 11 - 7 = 4$

$S_{25} = n/2\,[2a + (n-1) \times d]$
$S_{25} = \frac{25}{2}\,[2 \times 7 + (25-1) \times 4]$

$S_{25} = \frac{25}{2} [14 + 96]$

$S_{25} = \frac{25}{2} \times \cancel{110}^{55}$

$S_{25} = 1375$ Ans.

(ii) $a_n = 7 - 3n$ — ①

Ans. Substituting various values of n in eq-①

$a_1 = 7 - 3(1) = 4$

$a_2 = 7 - 3(2) = 1$

$a_3 = 7 - 3(3) = -2$

$d = 1 - 4 = -3$

$S_{25} = \frac{n}{2} [2a + (n-1) \times d]$

$S_{25} = \frac{25}{2} [2 \times 4 + (25-1) \times -3]$

$S_{25} = \frac{25}{2} [8 - 75 + 3]$

$S_{25} = \frac{25}{\cancel{2}} \times (\cancel{-64})^{-32}$

$S_{25} = -800$ Ans.

Q:5) Find the the sum of first 51 term of A.P. in which 2nd and 3rd term are 14 and 18 respectively.

Ans.) $a_2 = 14$; $a_3 = 18$

$d = 18 - 14 = 4$, $d = 4$.

$a_1 \Rightarrow a + d = 14$

$a + 4 = 14$, $a_1 = 10$.

$a = 14 - 4$

$S_n = \frac{n}{2} [2a + (n-1) \times d]$

$S_{51} = \frac{51}{2} [2 \times 10 + (51-1) 4]$

$S_{51} = {}^{51}/_2 [20 + 200]$

$S_{51} = \cancel{51/2}\ \frac{51 \times \cancel{220}^{110}}{\cancel{2}}$

$S_{51} = 5,610$ Ans.

Q:67) The first and last term of an A.P. are 17 and 350 respectively. Common difference is 9 then find the no. of term in A.P. & their sum.

Ans.) $a_1 = 17$; $l/a_n = 350$

$d = 9$

$a_n = a + (n-1) d$

$350 = 17 + (n-1) 9$

$\frac{350 - 17}{9} = n - 1$

$\frac{333}{9} = n - 1$

$n - 1 = 37$

$n = 37 + 1$

$n = 38$. Ans.

$S_n = {}^{n}/_2 [a + l]$

$S_{38} = \frac{\cancel{38}^{19}}{\cancel{2}} [17 + 350]$

$S_{38} = 19 \times 367$

$S_{38} = 6973$ Ans.

Q: 7) Find the sum of all odd no.s divisible by 3 between 1 and 1000.

Ans) Odd no.s divisible by 3 between 1 - 1000 are

$3, 9, 15, 21, \ldots\ldots 999.$

$a_1 = 3$; $d = 6$; $a_n / l = 999$

$$a_n = a + (n-1) \times d$$

$$999 = 3 + (n-1) \times 6$$

$$\frac{999-3}{6} = n-1$$

$$n-1 = \frac{996}{6}$$

$$n-1 = 166$$

$$n = 166+1$$

$$n = 167$$

$$S_n = \frac{n}{2}[2a + (n-1) \times d] \text{ or } \frac{n}{2}[a+l]$$

$$= S_{167} = \frac{167}{2}[3+999]$$

$$= S_{167} = \frac{167 \times \cancel{1002}^{501}}{\cancel{2}_1}$$

$$= S_{167} = 83,667 \text{ Ans.}$$

$n = 167$; ~~S_{167}~~ = Sum of odd no. divisible by 3 between 1-1000 = 83,667.

Q: 8) First term of A.P. is 8, n^{th} term is 33 and sum of first n term is 123, the find n & common difference.

Ans.) $a_1 = 8$; $a_n = 33$; $S_n = 123$

$a_n = a + (n-1) d$

$37 = 8 + (n-1) \times d$

$37 - 8 = (n-1) \times d$

$(n-1) d = 25$ ——— (1)

$S_n = n/2 \, [2a + (n-1) \times d]$

$123 = n/2 \, [2 \times 8 + 25]$ {from eq 1 $(n-1) \times d = 25$}

$123 = n/2 \, [16 + 25]$

$123 = \frac{n \times 41}{2}$

$\frac{123 \times 2}{41} = n$

$n = 246/41$

$n = 6$

Put $n = 6$ in eq (1) = $(n-1) d = 25$

$(6-1) d = 25$

$5d = 25$

$d = 25/5$

$d = 5$

$\therefore$, $n = 6$; $d = 5$ Ans.

Q:9) A sum of ₹ 280 is to be used to give four cash prize. If each prize is ₹ 20 less than its preceding prize Find the value of each of the prizes.

Ans.) Let, first prize is a_1 = ₹ a

second prize a_2 = ₹ $a - 20$

Third prize a_3 = ₹ $[(a-20)] - 20$ ⇒ ₹ $a - 40$

Fourt prize a_4 = ₹ $(a - 40) - 20$ ⇒ $a - 60$

Here, first term = a

$d = (a-20) - a = -20$; $n = 4$ (given)

$S_n = 280$ (given)

$S_n = n/2 \, [2a + (n-1)d]$

$280 = 4/2 \, [2a + (4-1) \times -20]$

$280 = 2 \, [2a - 60]$

$280 = 4a - 120$

$4a - 120 = 280$

$4a = 280 + 120$

$a = \frac{400}{4}$

$a = 100$ So first prize $a_1 = 100$

$a_2 = a - 20 = 100 - 20 = 80$

$a_3 = a - 40 = 100 - 40 = 60$

$a_4 = a - 60 = 100 - 60 = 40$

Series :- ₹100, ₹80, ₹60, ₹40.

Q:10 A manufature of T.V. sets produced 600 sets in 3rd year and 700 sets in the 7th year. Assuming that the production increases uniformly by a fixed no. every year, find:

Let production of 4 sets in Ist year = a

Given = $a_3 = 600$, $a_7 = 700$

$a_3 = a + 2d = 600$ — ①

$a_7 = a + 6d = 700$ — ②

Compare eq ① & ②

$-4d = -100$

$d = \frac{100}{4} = 25$

(i) The production in the 1st year.

Ans) 1st year = a,

$\Rightarrow$ To find a we put $d = 25$ in eq 1

$a + 2d = 600$

$a + 2 \times 25 = 600$

$a + 50 = 600$

$a = 600 - 50$

$a = 550$.

Thus, the production of 1st year is 550 _ Ans.

(ii) The production in the 10th year.

Ans.) 10th year = a_{10}

$a_n = a + (n-1)d$

$a_{10} = 550 + (10-1) \times 25$

$a_{10} = 550 + 9 \times 25$

$a_{10} = 550 + 225$

$a_{10} = 775$

Thus, the production of 10th year is 775 _ Ans.

(iii) The total production in 7th year.

Ans. $S_7 = ?$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

$S_7 = \frac{7}{2}[1100 + 150]$

$S_7 = \frac{7}{\cancel{2}_1} \times \cancel{1250}^{625}$

$S_7 = 4375$ Ans.

Thus, total production in first 7 years is 4375 Ans.